

# 01 Foundations

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If necessary, look up the relevant distribution from the course textbook (at the end of the book, there is a list of distributions.)

1. We plan to toss a fair coin three times.
  - What is the theoretical probability of obtaining
    - 0 heads
    - 1 heads
    - 2 heads
    - 3 heads
  - What is the theoretical probability of obtaining
    - 0 or 1 heads
    - 0,1, or 2 heads
    - 0-3 heads?
2. Toss a coin 10 times and count the number of heads and put a tick mark in the relevant column below. If you got four heads, for example, put a tick mark under 4. Then, compute, using pbinom, the theoretical probability of getting 0, 1, 2, . . . , 10 heads, assuming that the coin is fair. Hint: given sample size  $n$ , your assumed probability of a heads  $prob$ , and the number of heads you got  $x$ , the pbinom function delivers  $P(X \leq x)$ , the probability of getting  $x$  heads **or less**. In other words, pbinom is the cumulative distribution function.

Note that you have to compute  $P(X = x)$ !

|                          |   |   |   |   |   |   |   |   |   |   |    |
|--------------------------|---|---|---|---|---|---|---|---|---|---|----|
| Number of heads:         | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Theoretical probability: |   |   |   |   |   |   |   |   |   |   |    |
| Count:                   |   |   |   |   |   |   |   |   |   |   |    |

3. Given  $X \sim f(\cdot)$ , where  $f(\cdot)$  is (a)  $Unif(0, 10)$ , (b)  $N(\mu = 100, \sigma^2 = 20)$ , (c)  $Binom(p = .6, n = 20)$ . Find the probability of  $P(X < 3)$ ,  $P(X > 11)$ ,  $P(X = 6)$  for each distribution.

Fill in the table below.

| $f(\cdot)$       | Prob.         | Answer |
|------------------|---------------|--------|
| Uniform(0,10)    | $P(X < 3) =$  |        |
|                  | $P(X > 11) =$ |        |
|                  | $P(X = 6) =$  |        |
| Normal(100,20)   | $P(X < 3) =$  |        |
|                  | $P(X > 11) =$ |        |
|                  | $P(X = 6) =$  |        |
| Binom(p=.6,n=20) | $P(X < 3) =$  |        |
|                  | $P(X > 11) =$ |        |
|                  | $P(X = 6) =$  |        |

4. A random variable  $X$  comes from a LogNormal distribution with mean 6 log milliseconds and standard deviation 2 log ms. Plot this distribution. What is the probability  $Prob(X < 6)$  and  $Prob(X > 6)$ ? What is  $Prob(2 < X < 8)$ .
5. A random variable  $X$  comes from a Gamma distribution with mean 100 and standard deviation 10. Find the shape and rate parameters of the Gamma distribution.